

UNIVERSAL GROUPS AND GROUP STRUCTURES ON THE URYSOHN SPACE

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We present a separable abelian group \mathbb{G} equipped with invariant metric that is metrically universal for the class of all separable abelian groups with invariant metric, i.e. if H is a separable abelian group with invariant metric then there is an isometric isomorphism between H and a subgroup of \mathbb{G} . This answers a question of Shkarin.

We also show that \mathbb{G} is isometric to the Urysohn universal space and that this group structure on the Urysohn space is different from the previous known examples.

Finally, answering a question of Vershik whether there exists a non-abelian group structure on the Urysohn space we present a non-abelian metric group (having a free group of countably many generators as a dense subgroup) again isometric to the Urysohn space; with a yet unproven conjecture that this group is a metrically (or at least topologically) universal group for the class of all separable groups with two-sided invariant metric.